Inverse Design for Conditional Distribution Matching

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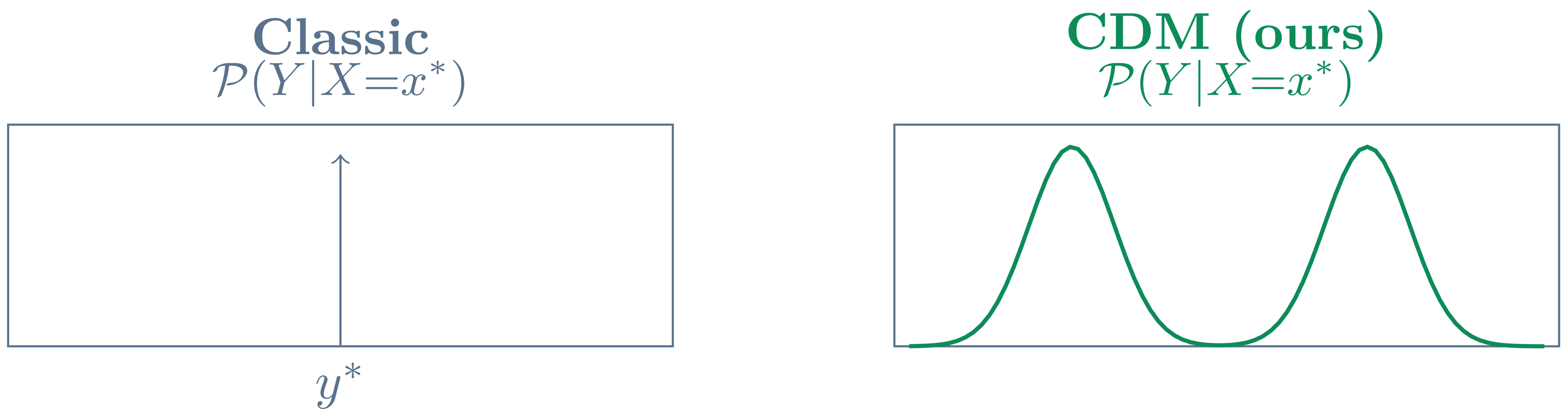
1. Motivation

Beyond Point Targets

Standard inverse design finds x^* such that $f(x^*) \approx y^*$. Many real design goals are **distributional**:

> Generative pipeline with outputs *balanced across demographic groups*



2. Problem Formulation

Conditional Distribution Matching (CDM)

Given joint distribution $\mathcal{P}(X,Y)$ and a user-specified target $\mathcal{G}(Y)$

Problem 1 — CDMS (Sampling)

Sample from the:

$$\mathcal{Q}_\beta(x) \propto \mathcal{P}(x)\, e^{-\beta\,\mathcal{L}(x)}, \quad \mathcal{L}(x) = \|\mathcal{P}(Y|X{=}x) - \mathcal{G}(Y)\|$$

When $\beta \geq 0$ trades off prior faithfulness vs. minimizing $\mathcal{L}$.

Problem 2 — CDMO (Optimization)

$$x^* = \arg\min_x \|\mathcal{P}(Y \mid X{=}x) - \mathcal{G}(Y)\|$$

Problem 2 is the $\beta{\to}\infty$ limit of Problem 1.

3. Method: MLGD-F

Matching-Loss Guided Diffusion with a Fast Sampler

Given pretrained model for $\mathcal{P}(X)$ (diffusion model with score s_θ) and few-step sampler $f_\phi(x,\eta)$ approximating $\mathcal{P}(Y \mid X{=}x)$.

Alg. 1 — Outer Loop (MLGD)

Input: $x_T \sim \mathcal{N}(0,I)$, score s_θ, sampler f_ϕ, targets $\mathcal{S}_\mathcal{G}$, step sizes ζ_t, noise schedule $\{\bar\alpha_t\}$

Output: optimized x_0^*

for $t = T, T{-}1, \ldots, 1$:

- $\hat{x}_0 \leftarrow \frac{1}{\sqrt{\bar\alpha_t}}(x_t + (1{-}\bar\alpha_t)\, s_\theta(x_t,t))$ — predict $\hat{x}_0$ (Tweedie)
- $\hat{\mathcal{L}} \leftarrow$ Alg.2$(\hat{x}_0, f_\phi, \mathcal{S}_\mathcal{G})$ — matching loss estimator
- $\nabla_{x_t}\hat{\mathcal{L}} \leftarrow$ autograd$(\hat{\mathcal{L}}, x_t)$ — backprop
- $x_{t-1} \leftarrow$ denoise$(x_t, s_\theta, t) - \zeta_t\, \nabla_{x_t}\hat{\mathcal{L}}$ — denoise step + correction

Alg. 2 — Inner Estimator (distribution matching loss)

Input: $\hat{x}_0$, sampler f_ϕ, targets $\mathcal{S}_\mathcal{G}$

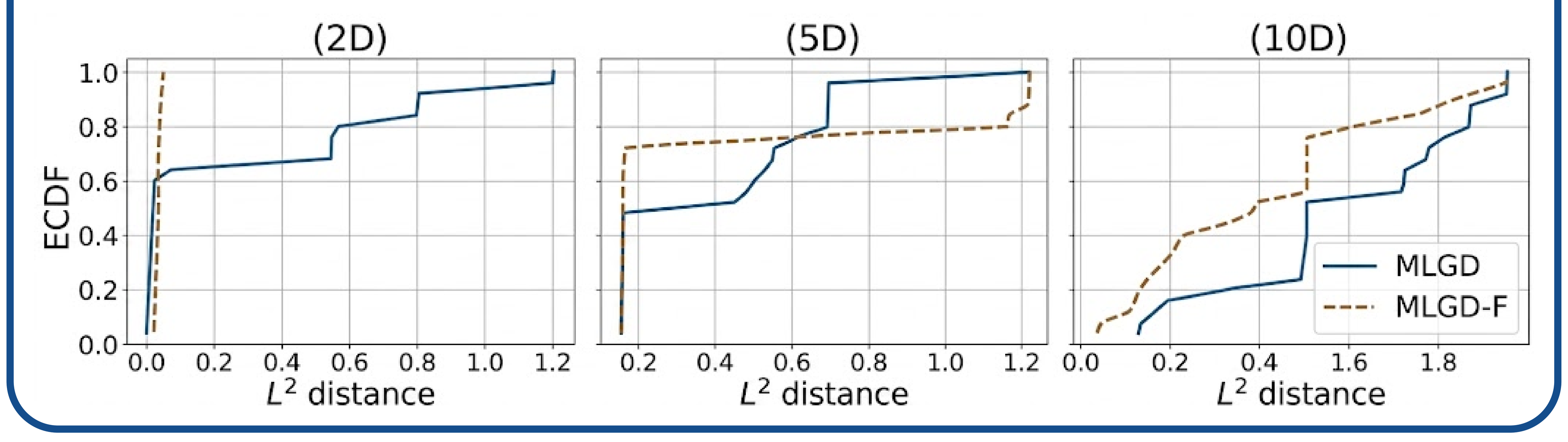
Output: $\hat{\mathcal{L}}$

- Draw $\eta_1,\ldots,\eta_{n_c} \sim \pi$ — n_c noise samples
- $\mathcal{S}_c \leftarrow \{f_\phi(\hat{x}_0,\eta_i)\}$ — conditional samples via f_ϕ
- **return** $\mathcal{L}(\mathcal{S}_c, \mathcal{S}_\mathcal{G})$ — e.g. MMD

When Alg. 2 uses a full diffusion chain the method is **MLGD**; when it uses a fast sampler - **MLGD-F**, where F stands for fast.

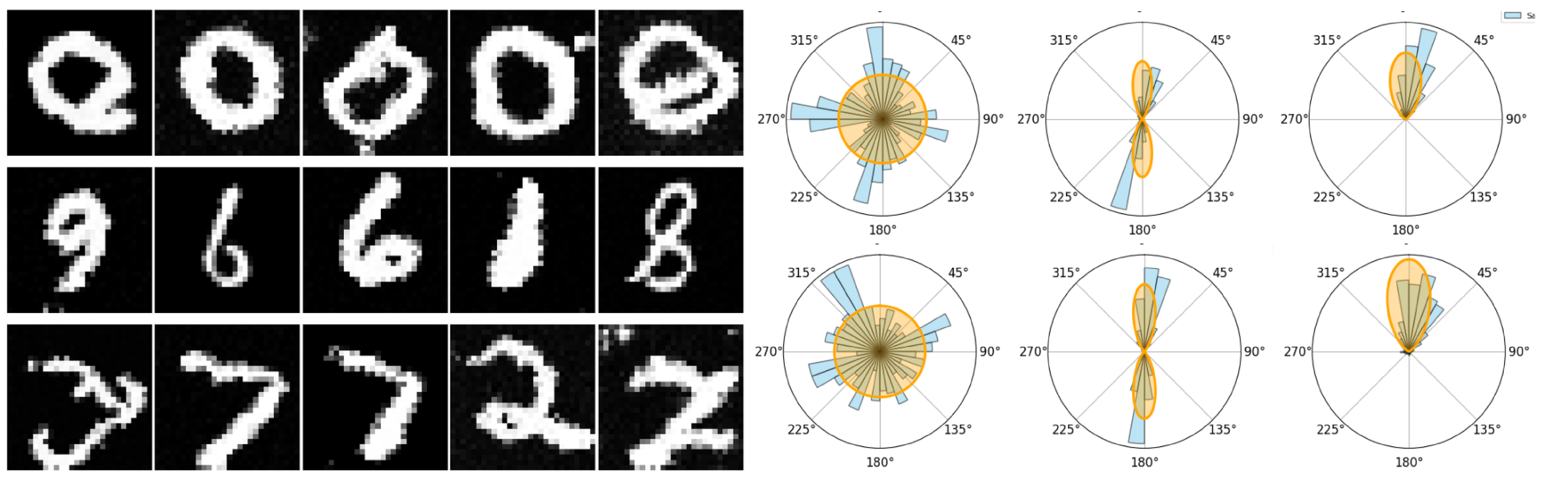
4. Scaling with Dimension: MLGD vs. MLGD-F

Despite MLGD's higher fidelity, its full chain accumulates gradient variance; MLGD-F yields better signal (see plots), 9×–14.8× speedup (2D–10D), and cuts peak VRAM to **43 GB** from projected 375 GB (Exp. 2).



5. Experiment 1: MNIST Rotation

$X \in \mathbb{R}^{784}$: rotated MNIST images; $Y \in \mathbb{R}^2$: rotation angle (polar); $\mathcal{G}$: digit-valid angle distribution. DDPM as $\mathcal{P}(X)$, Consistency model as f_ϕ.

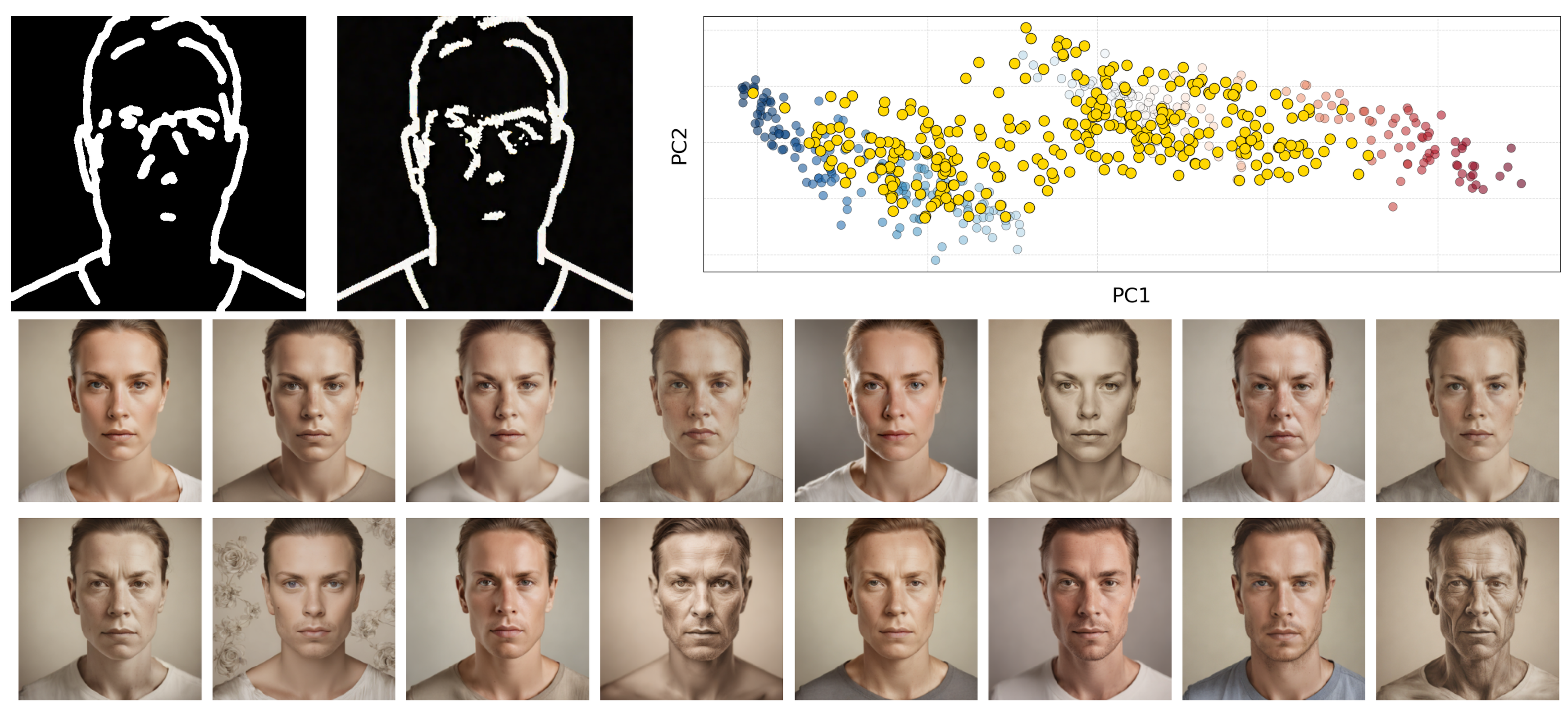


Left: Top-5 x^* by final denoise step $\hat{\mathcal{L}}$ (Alg. 2) per $\mathcal{G}$: uniform, bimodal, unimodal (top-bottom). **Right:** Top-2 x^*-f_ϕ vs. $\mathcal{G}$ (left-right, same order).

MLGD-F recovers **semantically meaningful** digits from rotation geometry: e.g., a uniform target yields **0**s, the only digit valid in all angles.

6. Experiment 2: Scribble-Conditioned Image

$X \in \mathbb{R}^{512\times512}$: scribble image; $Y \in \mathbb{R}^{768}$: CLIP embedding of scribble-conditioned image; $\mathcal{G}$: gender/age distribution. Stable Diffusion XL (SDXL) as $\mathcal{P}(X)$, SDXL-Turbo conditioned on scribble $\hat{x}_0$ as f_ϕ



Top: source scribble (left); optimized x^* (middle); CLIP PCA (right): • male→• female = target $\mathcal{S}_\mathcal{G}$ (gender interpolation); • = interpolated images generated conditioned on x^*. **Bottom:** • images ordered female→male (PC1).

Scenario	MMD Improvement (↑)
MMD: kernel-based distribution distance (↓ better)	
Balanced (50% male)	+32.5%
Skewed (25% male)	+27.1%
Gender interpolation	+22.1%
Age interpolation	+28.5%

Improvements over the **source** (male), where conditioned images are ∼ 100% male. For balanced/skewed targets, MLGD-F achieves 47.4% male (target 50%, CI [45.3, 49.5]) and 26.4% male (target 25%, CI [24.5, 28.3]).

7. Conclusion

Summary

- ✓ First problem class to generalise inverse design to **distributional targets**
- ✓ **MLGD-F** operates at inference time with **no retraining or fine-tuning**
- ✓ Quality improves as stronger pretrained models become available

Limitations

- – Quality bounded by model f_ϕ fidelity
- – Requires end-to-end differentiable f_ϕ
- – Runtime may be prohibitive for latency-sensitive applications