

TOTAL PERFORMANCE EVALUATION OF INTENSITY ESTIMATION AFTER DETECTION AND ITS APPLICATION ON ACCUMULATED RAINFALL ESTIMATION

Taer Weiss

Under the supervision of Prof. Hagit Messer and Prof. Tirza Routtenberg

INTRODUCTION

ABSTRACT

- o In this work we focus on problems of estimation after detection where first, a signal is detected, and then, its parameters of interest are estimated.
- o We present a new performance evaluation measure that incorporates the estimation and detection-related errors under one risk function, and that is suitable for parameters representing intensity.
- o We derive a new Cramér-Rao type bound on the risk and present the use of the risk for the Pareto-efficient design of the detection threshold.
- o We apply our performance evaluation measure on the task of accumulated rainfall estimation from attenuation measurements of signals from commercial microwave links (CMLs).

Problem Formulation

$\mathcal{H}_0: \mathbf{x} \sim f_0(\mathbf{x}; \theta_0, \psi_0), \theta_0 \in [0, \theta_0]$,

$\mathcal{H}_1: \mathbf{x} \sim f_1(\mathbf{x}; \theta_1, \psi_1), \theta_1 \in [0, \theta_1]$

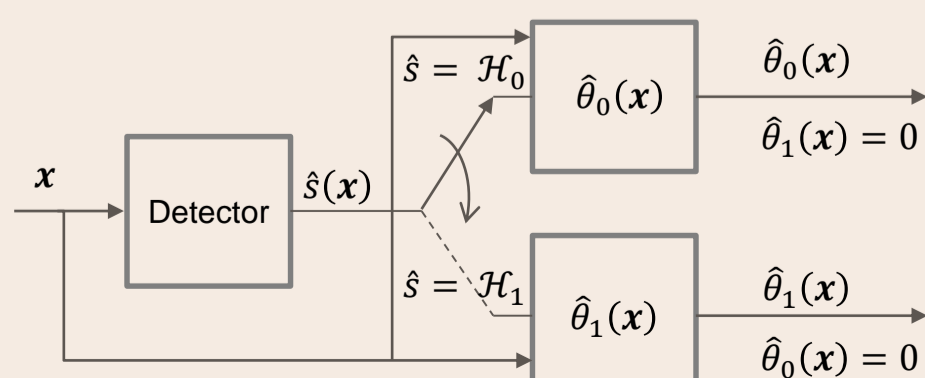
$\mathbf{x} \sim \Omega_x$ - the observation vector

$\theta_i, i = 0, 1$ - unknown deterministic parameters, represent the intensity of the related physical phenomenon.

θ_{1-i} does not exist under hypothesis \mathcal{H}_i and since it represents the intensity of a non-existing phenomenon, it can be treated as having zero value.

$\psi_i, i = 0, 1$ - known parameter vectors

Assumption: $f_0 \neq f_1$ and/or $\psi_0 \neq \psi_1$



RESEARCH

Intensity Estimation after Detection Risk

- o $MSSE_i(\hat{\theta}_i, \theta_i, \hat{s}) = P_i(\hat{s} = \mathcal{H}_i; \theta_i) \frac{E_i((\hat{\theta}_i - \theta_i)^2 | \hat{s} = \mathcal{H}_i; \theta_i)}{\theta_i^2}$
- o $Risk_i^{FA}(\hat{\theta}_{1-i}, \theta_i, \hat{s}) = P_i(\hat{s} = \mathcal{H}_{1-i}; \theta_i) \frac{E_i(\hat{\theta}_{1-i}^2 | \hat{s} = \mathcal{H}_{1-i}; \theta_i)}{\theta_{1-i}^2}$
- o $Risk_i^{MD}(\theta_i, \hat{s}) = P_i(\hat{s} = \mathcal{H}_{1-i}; \theta_i) \frac{\theta_i^2}{\theta_i^2}$

$\mathcal{R}(\hat{\theta}_0, \hat{\theta}_1, \theta_0, \theta_1, \hat{s}, \gamma) =$

$$\sum_{i=0}^1 (MSSE_i(\hat{\theta}_i, \theta_i, \hat{s}) + Risk_i^{FA}(\hat{\theta}_{1-i}, \theta_i, \hat{s}) + Risk_i^{MD}(\theta_i, \hat{s}))$$

Cramér-Rao Type Bound

$$b_i \triangleq E_i(\hat{\theta}_i - \theta_i | \hat{s} = \mathcal{H}_i; \theta_i), \quad b_i^* \triangleq E_i(\hat{\theta}_{1-i} | \hat{s} = \mathcal{H}_{1-i}; \theta_i)$$

$$V_i \triangleq \frac{\partial}{\partial \theta_i} \log f(\mathbf{x} | \hat{s} = \mathcal{H}_i; \theta_i), \quad J_i = E_i(V_i^2 | \hat{s} = \mathcal{H}_i; \theta_i)$$

$$U_i \triangleq \frac{\partial}{\partial \theta_i} \log f(\mathbf{x} | \hat{s} = \mathcal{H}_{1-i}; \theta_i), \quad J_i^* = E_i(U_i^2 | \hat{s} = \mathcal{H}_{1-i}; \theta_i)$$

- o Lower bound on the MSSE:

$$MSSE_i(\hat{\theta}_i, \theta_i, \hat{s}) \geq \frac{P_i(\hat{s} = \mathcal{H}_i; \theta_i)}{\theta_i^2} B_{MSSE_i}$$

$$B_{MSSE_i} \triangleq \left(1 + \frac{\partial b_i}{\partial \theta_i}\right)^2 J_i^{-1} + b_i^2$$

- o Lower bound on the FA risk:

$$Risk_i^{FA}(\hat{\theta}_{1-i}, \theta_i, \hat{s}) \geq \frac{P_i(\hat{s} = \mathcal{H}_{1-i}; \theta_i)}{\theta_{1-i}^2} B_{Risk_i^{FA}}^*$$

$$B_{Risk_i^{FA}}^* \triangleq \left(\frac{\partial b_i^*}{\partial \theta_i}\right)^2 J_i^{*-1} + b_i^{*2}$$

Pareto Efficiency

$$\mathcal{R}(\hat{\theta}_0, \hat{\theta}_1, \theta_0, \theta_1, \hat{s}, \gamma) = \mathcal{R}(\gamma) = f_{est}(\gamma) + f_{det}(\gamma)$$

$$f_{est}(\gamma) = \sum_{i=0}^1 MSSE_i(\gamma), f_{det}(\gamma) = \sum_{i=0}^1 (Risk_i^{FA}(\gamma) + Risk_i^{MD}(\gamma))$$

$$\gamma_0 = \arg \min_{\gamma \in \Gamma} \mathcal{R}(\gamma) \rightarrow$$

γ_0 is a Pareto Efficient solution w.r.t the estimation and detection objective functions

Simulation

$$\mathcal{H}_0: x[n] = \alpha \theta_0 + \omega_0[n] \quad n = 0, \dots, N-1$$

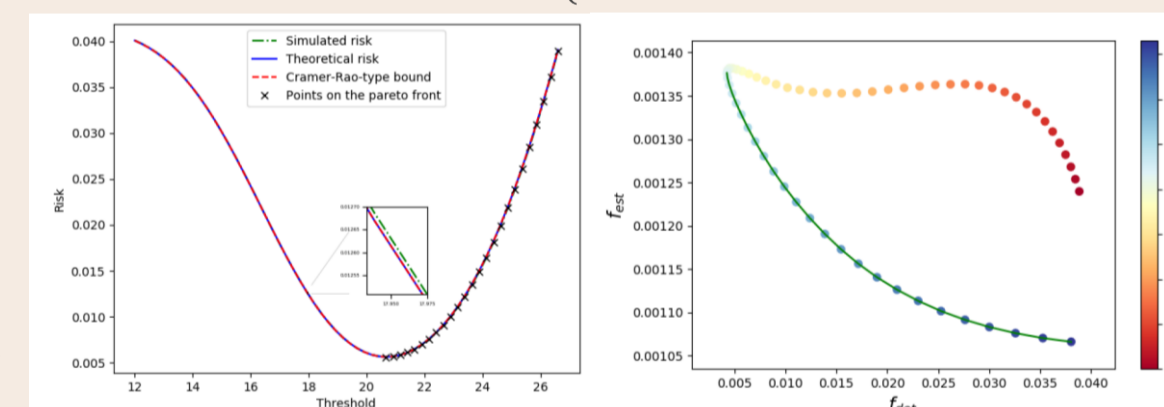
$$\mathcal{H}_1: x[n] = \beta \theta_1 + \omega_1[n]$$

α, β are known system parameters

ω_0, ω_1 are zero-mean Gaussian noise sequences with $\sigma_0^2 \neq \sigma_1^2$

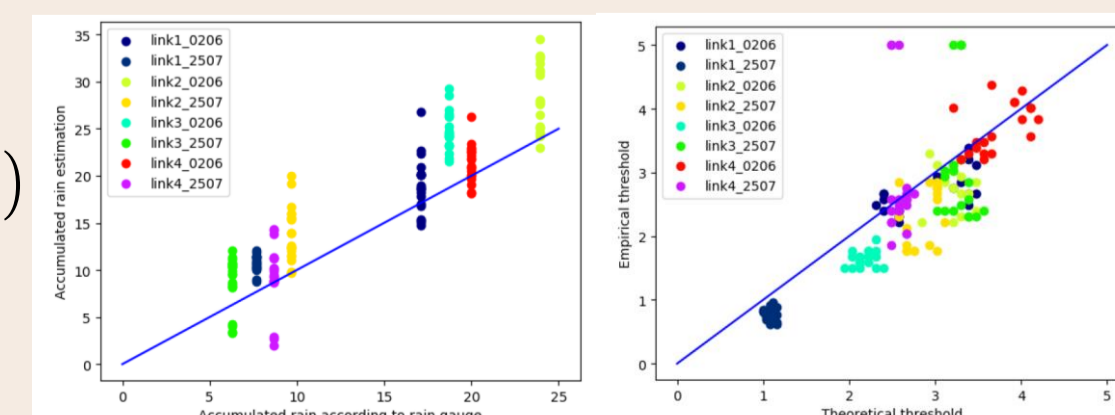
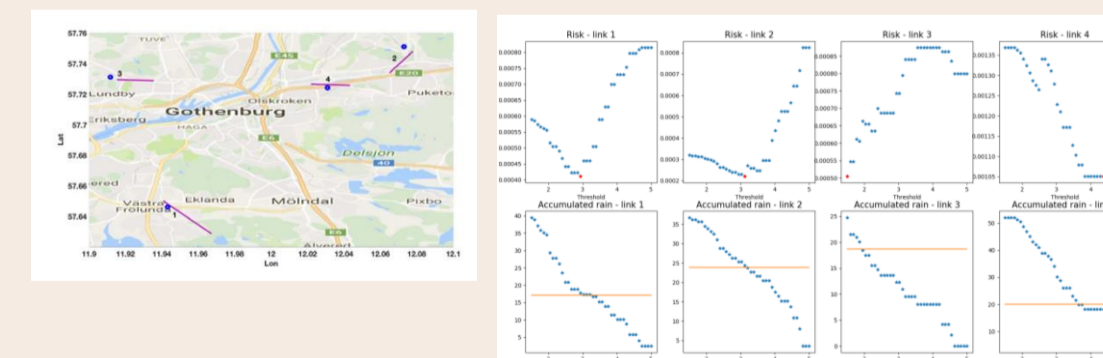
$$\hat{s} = \begin{cases} \mathcal{H}_1, & \text{if } \frac{1}{N} \sum_{n=0}^{N-1} x[n] \geq \gamma \\ \mathcal{H}_0, & \text{otherwise} \end{cases} \quad \hat{\theta}_1 = \begin{cases} \min \left(\max \left(\frac{1}{\beta N} \sum_{n=0}^{N-1} x[n], 0 \right), \theta_1 \right), & \text{if } \hat{s} = \mathcal{H}_1 \\ 0, & \text{if } \hat{s} = \mathcal{H}_0 \end{cases}$$

$$\hat{\theta}_0 = \begin{cases} \min \left(\max \left(\frac{1}{\alpha N} \sum_{n=0}^{N-1} x[n], 0 \right), \theta_0 \right), & \text{if } \hat{s} = \mathcal{H}_0 \\ 0, & \text{if } \hat{s} = \mathcal{H}_1 \end{cases}$$



Pareto Efficient points are marked with X in the left figure and with the green line in the right figure.

Accumulated Rainfall Estimation Using the Risk



CONCLUSION

CONCLUSIONS

- o We presented a new performance evaluation measure for estimation after detection of intensity values, that properly incorporates both estimation and detection-related errors.
- o We developed a Cramér-Rao type bound on the risk, and showed that the minimization of the risk results in a Pareto Efficient solution w.r.t the estimation and detection objective functions.
- o We utilized the proposed risk in the task of accumulated rainfall estimation from signal attenuation measurements from CMLs, as a system design tool for setting the detection threshold.
- o We showed that the empirical threshold that minimizes the risk has a good resemblance to the theoretical threshold that best matches the estimated accumulated rain with the value measured by the corresponding rain gauge.

PAPERS

Taer Weiss, Tirza Routtenberg, and Hagit Messer. "Total performance evaluation of intensity estimation after detection." Signal Processing 183 (2021)

Taer Weiss, Tirza Routtenberg, Jonatan Ostrometzky, and Hagit Messer. "Intensity Estimation After Detection for Accumulated Rainfall Estimation" - Preprint

CONTACT INFO

taerwe@gmail.com
messer@eng.tau.ac.il
tirzar@bgu.ac.il